1 Fractions

$$\frac{\frac{a}{b}}{c} = \frac{a}{b \cdot c} \qquad \text{a} \div \frac{b}{c} = \frac{a \cdot c}{b} \qquad a \cdot \frac{b}{c} = \frac{a \cdot b}{c} \, not\ mixed\ fraction$$

$$\frac{a}{b} = \frac{a}{c} \cdot \frac{c}{b} \qquad \frac{a}{b} - \frac{c}{d} = \frac{a \cdot d - b \cdot c}{b \cdot d}$$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} \qquad \frac{a}{b} \div \frac{c}{d} = \frac{a \cdot d}{b \cdot c} = \frac{a}{b} \cdot \frac{d}{c}$$

2 Logarithms

$$\log_a x = \log_b x \cdot \frac{\log_b a}{\log_b b}, \qquad \ln e^x = x, \tag{1}$$

$$\text{if } x < y \quad \text{and} \quad a \geq 1, \qquad \log_a x < \log_a y$$

$$\text{if } 0 < a < 1, \qquad \log_a x > \log_a y$$

$$a^x = b \implies x = \log_a b, (a > 0, b > 0, a \neq 1) \tag{2}$$

$$\log_a^k b = \frac{1}{k} \log_a b \tag{3}$$

3 Powers and Roots

$$a^b \cdot a^c = a^{b+c} \qquad 2x^{\frac{-1}{2}} = \frac{2}{\sqrt{x}} \qquad \sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$$

$$x^{-\frac{1}{2}} = \frac{1}{\sqrt{x}} \qquad \sqrt[n]{x} = x^{\frac{1}{n}} \qquad \sqrt[n]{x} = x^{\frac{1}{n}}$$

$$\sqrt[m]{x}^n = x^{\frac{n}{m}} \qquad \frac{a^x}{a^y} = a^{x-y}$$

4 Factoring

$$(a+b)^2 = a^2 + 2ab + b^2 = (a+b)(a+b) \qquad (a-b)^2 = a^2 - 2ab + b^2 = (a-b)(a-b)$$

$$(a+b)(a-b) = a^2 - b^2 \qquad (a-b)(a^2 + ab + b^2) = a^3 - b^3$$

$$(a+b)(a^2 - ab + b^2) = a^3 + b^3 \qquad (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

5 Trigonometry

$$\text{coordinates } (x, y)\ , \qquad \cos = x, \sin = y, \tan = \frac{y}{x} \tag{4}$$

$$\sin(\theta) = \frac{opposite}{hypotenuse}, \qquad \csc(\theta) = \frac{hypotenuse}{opposite} = \frac{1}{\sin(\theta)}$$

$$\cos(\theta) = \frac{adjacent}{hypotenuse}, \qquad \sec(\theta) = \frac{hypotenuse}{adjacent} = \frac{1}{\cos(\theta)}$$

$$\tan\theta = \frac{opposite}{adjacent}, \qquad \cot\theta = \frac{adjacent}{opposite} = \frac{1}{\tan\theta}$$

reference angle in each quadrant

I θ II $\pi - \theta$ III $\theta - \pi$ IV $2\pi - \theta$, tan is not defined for multiples of pi/2, period every pi*k (5)

Positive: All (1) Students (2) Take (3) Calculus (4) $=$ all, sin, tan, cos Soh Cah Toa (6)

(7)

$$\tan(-\theta) = -\tan(\theta) \qquad \sin(-\theta) = -\sin(\theta) \qquad \cos(\pi + \theta) = -\cos(\theta)$$

$$\sin(\pi + \theta) = -\sin(\theta) \qquad \cos(\pi - \theta) = -\cos(\theta) \qquad \sin(\pi - \theta) = \sin(\theta)$$

$$\cos(\frac{\pi}{2} + \theta) = -\sin(\theta) \qquad \sin(\frac{\pi}{2} + \theta) = \cos(\theta) \qquad \cos(\frac{\pi}{2} - \theta) = \sin(\theta)$$

$$\sin(\frac{\pi}{2} - \theta) = \cos(\theta)$$

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 - \cos^2(a) = \sin^2(a) \qquad 1 - \sin^2(a) = \cos^2(a)$$

$$\cot^2\theta + 1 = \frac{1}{\sin^2\theta}$$

$$\text{Period } T = \frac{2\pi}{k}\ , \qquad \text{Amplitude } A = \frac{y_{max} - y_{min}}{2}\ , \qquad \text{Phase Shift } d = \frac{2\pi}{k}\ ,$$

$$\text{Vertical Shift } c = \frac{y_{max} + y_{min}}{2}$$

a = amplitude and reflection in the x-axis if negative

k = period of fn, P = std period(360=2pi)/k

c = vertical shift - determine eqn of axis of curve y=c d = phase shift - left - or right +

a*sin(k(x-d)) + c

6 General

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\ , \qquad where\ ax^2 + bx + c = 0$$

$$b^2 - 4ac = \text{discriminant}\ , \qquad \text{if } b^2 - 4ac < 0 \text{ then no roots}$$

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$|y| < a,\ -a < y < a \qquad |y| > a,\ y > a \text{ or } y < -a$$

Even $f(x) = f(-x)$, Odd $f(-x) = -f(x)$ sign changes at odd roots, but not even ones

one to one injective - no horizontal line crosses more than once

onto surjective- horizontal line crosses at least once

7 Conic Sections

$$\text{Ellipse } \frac{x^2 - x_0}{a^2} + \frac{y^2 - y_0}{b^2} = 1$$

$$\text{Center } (x_0, y_0) \text{ Foci } (x_0 \pm a, y_0) \text{ Vertices } (x_0 \pm a, y_0 \pm b)$$

$$\text{major axis = 2a, minor axis = 2b , } (a > b) \text{Hyperbola } \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\text{asym } y = \frac{-a}{b}x, x \ = \frac{a}{b}x$$

$$\text{Parabola } y = ax^2 + bx + c$$

$$\text{Circle } x^2 + y^2 = r^2$$

$$\text{Distance of Circle } d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = r$$

$$\text{Distance of Point to Line } d = \frac{|Ax + By + c|}{\sqrt{A^2 + B^2}}$$

8 Polynomials

$$x^2 + 2ax + a^2 - b = (x + a)^2 - b$$

$$\text{Complete Square } x^2 + bx + (\frac{b}{2})^2 = (x + \frac{b}{2})^2$$

9 Function Transformations

$$\text{Vertical Shift } f(x) = g(x) + k \text{ where } k \text{ is the shift}$$

$$\text{Horizontal Shift } f(x) = g(x - k) \text{ where } k \text{ is the shift}$$

$$\text{Reflection } f(x) = -g(x) \text{ or } f(x) = g(-x)$$

$$\text{Stretch } f(x) = g(ax) \text{ or } f(x) = g(x/a)$$

$$\text{Combinations } f(x) = g(ax + b) \text{ or } f(x) = g(x/a + b)$$

$$\text{Reflection } f(x) = g(-x) \text{ or } f(x) = g(x) \text{ if } g(x) \text{ is even}$$

$$\text{Vertical Asymptote } f(x) = \frac{1}{x} \text{ at } x = 0$$

$$\text{Horizontal Asymptote } f(x) = \frac{1}{x} \text{ at } y = 0$$

$$\text{Domain } f(x) = \frac{1}{x} \text{ is } x \text{ not equal to } 0$$

$$\text{Range } f(x) = \frac{1}{x} \text{ is } y \text{ not equal to } 0$$

10 Set Theory

$$A \setminus B = \{x \in A \mid x \notin B\} = A \cap \overline{B}$$

Associativity $A \cap (B \cap C) = (A \cap B) \cap C \; , \; A \cup (B \cup C) = (A \cup B) \cup C$

Commutativity $A \cap B = B \cap A \; , \; A \cup B = B \cup A, (B \cup C) \cap A = (B \cap A) \cup (C \cap A)$

Distributivity $A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \; , \; A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

De-morgan $\overline{(A \cap B)} = \overline{A} \cup \overline{B} \; , \; \overline{(A \cup B)} = \overline{A} \cap \overline{B}$

more $A \setminus (B \setminus C) = (A \setminus B) \cup (A \cap C)$

Union $A \cup \emptyset = A \; , \; A \cap \emptyset = \emptyset$

11 Summations

Sum of first n terms of sequence

$$\sum_{i=1}^{n} a_i = \frac{n(n+1)}{2}$$

n is the number of terms,a1 is first term, ai is ith term, an is last term

for arithmetic sequences, d = difference between terms

for geometric sequences, r = common ratio

Sum of 2 sums $\sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i = \sum_{i=1}^{n} (a_i + b_i)$

Sum of a sum and constant $\sum_{i=1}^{n} a_i + c = \sum_{i=1}^{n} (a_i + c)$

sin is odd, cos is even, tan is neither

even powers are even, odd powers are odd

absolute value is even

log is neither, e

positive base to negative power is

12 Limits

$$\lim_{x \to \infty} arctan(x) = \frac{\pi}{2} \qquad \lim_{x \to -\infty} arctan(x) = -\frac{\pi}{2}$$

$\lim_{x \to a^+} f(x) = L$ and $\lim_{x \to a^-} f(x) = L$ then $\lim_{x \to a} f(x) = L$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$