Formula Sheet

בס"ד

I – Sentential Logic

LOGICAL IMPLICATIONS גרירות לוגיות	
$R \Rightarrow R \vee S$	: I1
$S \Rightarrow R \vee S$	: I2
$R \wedge S \Rightarrow R$	: I3
$R \wedge S \Rightarrow S$	: I4
R and $S \Rightarrow R \wedge S$	: I5
$\neg R$ and $(R \vee S) \Rightarrow S$	: I6
$S \Rightarrow (R \rightarrow S)$	: I7
$\neg R \Rightarrow (R \rightarrow S)$	: I8
$\neg(R \rightarrow S) \Rightarrow R$	: I9
$\neg(R \rightarrow S) \Rightarrow \neg S$	: I10
R and $(R \rightarrow S) \Rightarrow S$	: I11
$\neg S$ and $(R \rightarrow S) \Rightarrow \neg R$	: I12
$(P \rightarrow R)$ and $(R \rightarrow S) \Rightarrow P \rightarrow S$	I13
$(P \vee R)$ and $(P \rightarrow S)$ and $(R \rightarrow S) \Rightarrow S$	I14

EQUIVALENCES שקילויות	
$R \vee T \Leftrightarrow T$	: E1
$R \vee F \Leftrightarrow R$	: E2
$R \wedge F \Leftrightarrow F$	: E3
$R \wedge T \Leftrightarrow R$	: E4
$R \vee R \Leftrightarrow R$	: E5
$R \wedge R \Leftrightarrow R$	: E6
$R \vee (\neg R) \Leftrightarrow T$	: E7
$R \wedge (\neg R) \Leftrightarrow F$	: E8
$\neg(\neg R) \Leftrightarrow R$	: E9
$R \vee S \Leftrightarrow S \vee R$	: E10
$R \wedge S \Leftrightarrow S \wedge R$	: E11
$R \vee (S \vee Q) \Leftrightarrow (R \vee S) \vee Q$	: E12
$R \wedge (S \wedge Q) \Leftrightarrow (R \wedge S) \wedge Q$	: E13
$R \vee (S \wedge Q) \Leftrightarrow (R \vee S) \wedge (R \vee Q)$	: E14
$R \wedge (S \vee Q) \Leftrightarrow (R \wedge S) \vee (R \wedge Q)$	: E15
$\neg(R \vee S) \Leftrightarrow \neg R \wedge \neg S$	: E16
$\neg(R \wedge S) \Leftrightarrow \neg R \vee \neg S$	: E17
$R \vee (R \wedge S) \Leftrightarrow R$	: E18
$R \wedge (R \vee S) \Leftrightarrow R$	: E19
$R \rightarrow S \Leftrightarrow \neg R \vee S$	: E20
$R \rightarrow S \Leftrightarrow \neg S \rightarrow \neg R$	: E21
$R \leftrightarrow S \Leftrightarrow (R \rightarrow S) \wedge (S \rightarrow R)$	: E22
$R \rightarrow (S \rightarrow Q) \Leftrightarrow (R \wedge S) \rightarrow Q$	: E23

II – Predicate Logic

LOGICAL IMPLICATIONS גרירות לוגיות	
$\exists x(P(x) \wedge Q(x)) \Rightarrow \exists x P(x) \wedge \exists x Q(x)$	:I15
$\exists x P(x) \rightarrow \exists x Q(x) \Rightarrow \exists x(P(x) \rightarrow Q(x))$	:I16
$\exists x\ P(x) \rightarrow \forall x\ Q(x) \Rightarrow \forall x\ (P(x) \rightarrow Q(x))$	:I17
$\forall x P(x) \Rightarrow \exists x P(x)$	:I18
$\forall x P(x) \vee \forall x Q(x) \Rightarrow \forall x(P(x) \vee Q(x))$	: I19
$\forall x\ (P(x) \rightarrow Q(x)) \Rightarrow \forall x\ P(x) \rightarrow \forall x\ Q(x)$	:I20
$\forall x\ (P(x) \rightarrow Q(x)) \Rightarrow \exists x\ P(x) \rightarrow \exists x\ Q(x)$	:I21
$\forall x\ P(x) \rightarrow \forall x\ Q(x) \Rightarrow \exists x\ (P(x) \rightarrow Q(x))$	:I22
$\exists x\ \forall y\ P(x,y) \Rightarrow \forall y\ \exists x\ P(x,y)$	:I23
$\forall x\ \exists y\ (P(x,y) \rightarrow Q(x,y))$ $\Rightarrow \exists x\ \forall y\ P(x,y) \rightarrow \exists x\ \exists y\ Q(x,y)$	:I24

EQUIVALENCES שקילויות	
$\exists x\, P(x) \Leftrightarrow \neg \forall x\ \neg P(x)$	: E24
$\forall x\, P(x) \Leftrightarrow \neg \exists x\ \neg P(x)$	: E25
$\exists x (P(x) \vee Q(x)) \Leftrightarrow \exists x\, P(x) \vee \exists x\, Q(x)$	: E26
$\forall x\ (P(x) \wedge Q(x)) \Leftrightarrow \forall x\ P(x) \wedge \forall x\ Q(x)$	: E27
$\exists x\ \exists y\ P(x,y) \Leftrightarrow \exists y\ \exists x\ P(x,y)$	:E28
$\forall x\ \forall y\ P(x,y) \Leftrightarrow \forall y\ \forall x\ P(x,y)$	: E29
if x is not free in Q $\exists x\ (P(x) \vee Q) \Leftrightarrow \exists x\ P(x) \vee Q$	:E30
if x is not free in P $\exists x(P \vee Q(x)) \Leftrightarrow P \vee \exists x Q(x)$	: E31
if x is not free in Q $\exists x(P(x) \wedge Q) \Leftrightarrow \exists x P(x) \wedge Q$	: E32
אם x איננו חופשי ב- P $\exists x(P \wedge Q(x)) \Leftrightarrow P \wedge \exists x\, Q(x)$	: E33
if x is not free in Q $\forall x(P(x) \vee Q) \Leftrightarrow \forall x\ P(x) \vee Q$	: E34
if x is not free in P $\forall x\ (P \vee Q(x)) \Leftrightarrow P \vee \forall x\ Q(x)$	:E35
if x is not free in Q $\forall x(P(x) \wedge Q) \Leftrightarrow \forall x P(x) \wedge Q$	:E36
if x is not free in P $\forall x(P \wedge Q(x)) \Leftrightarrow P \wedge \forall x\, Q(x)$	: E37
(*) $Q_1 x P(x) \vee Q_2 y R(y) \Leftrightarrow Q_1 x Q_2 y (P(x) \vee R(y))$	: E38
(*) $Q_1 x P(x) \wedge Q_2 y R(y) \Leftrightarrow Q_1 x Q_2 y (P(x) \wedge R(y))$	: E39

(*) In E38 and E39, Q_1 and Q_2 are quantifiers.

The Deductive System L_2

Axiom $\alpha \vee \neg\alpha$

Rules of Inference :

R1 Premise

R2 Equivalence or implication

R3 Deduction Theorem

R4 Proof by contradiction

R5 Given $\forall x\varphi(x)$, we can write $\varphi(t)$, where t is some variable or constant.

R6 Given $\exists x\varphi(x)$, we can write $\varphi(t)$, where t is some variable or constant such that:

1. t is a (constant/variable) that does not (appear/appear free) in any premise.
2. t is a (constant/variable) that does not (appear/appear free) in any previous lines of the deduction.

R7 Given $\varphi(t)$, we can write $\exists x\varphi(x)$, where t is some variable or constant.

R8 Given $\varphi(t)$, we can write $\forall x\varphi(x)$, on condition that t is some variable or constant such that:

1. t is a (constant/variable) that does not (appear/appear free) in any premise.
2. t is a (constant/variable) that does not (appear/appear free) in any previous lines of the deduction obtained by R6.

In **R5**-**R8**, t is a variable or constant that is subsitutable for x in $\varphi(x)$

The Deductive System $L_{\rightarrow}$

Axioms :

A1 $\alpha \rightarrow (\beta \rightarrow \alpha)$

A2 $(\alpha \rightarrow (\beta \rightarrow \gamma)) \rightarrow ((\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \gamma))$

A3 $(\neg\beta \rightarrow \neg\alpha) \rightarrow ((\neg\beta \rightarrow \alpha) \rightarrow \beta))$

Rules of Inference Modus Ponens.
Use of the deduction theorem is allowed.

Graph Theory

Euler's formula $n + f - m = 2$